



The Fuzzy Revolution: Goodbye to the Aristotelian Weltanschauung

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Because of its unorthodoxy, it has been and will continue to be controversial for some time. Eventually, though, the theory of fuzzy sets is likely to be recognized as a natural development in the evolution of scientific thinking. In retrospect, the skepticism about its usefulness will be viewed as a manifestation of the human attachment to tradition and resistance to innovation ([91], p. 421).

1. Dear Professor Zadeh

The medical artificial intelligence community congratulates you on your 80th birthday on 4 February 2001. On behalf of all patients and non-patients who are, or will in the future be, enjoying the medical fruits of your work, we would like to thank you for initiating and advancing *The Fuzzy Revolution* in science, technology, and society. Medicine has been among the privileged areas to early recognize this revolution and to embrace the fuzzy theory in parallel with the seminal Mamdani and Assilian application in the engineering sciences and technology [35,36] (see, e.g. [2,30,73,74]).¹ The tempo, the scope, and the quantity of medical fuzzy research and technology have exponentially increased in the meantime. There is no medical subdomain left outside this accelerating fuzzy stream today. This advancement of medical thinking and practice we owe to your foresight already

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¹ In the present context, the term ‘fuzzy theory’ is used as a unifying label for the theory of fuzzy sets + theory of linguistic variables and hedges + fuzzy logic + concomitant conceptual frameworks and theories based on these foundations. Although the term ‘fuzziness theory’ would be preferable, it may not be generally acceptable.

communicated in 1962, 3 years before the explicit inception of your fuzzy theory in 1965 [77,78]:

In fact, there is a fairly wide gap between what might be regarded as ‘animate’ system theorists and ‘inanimate’ system theorists at the present time, and it is not at all certain that this gap will be narrowed, much less closed, in the near future. There are some who feel this gap reflects the fundamental inadequacy of the conventional mathematics — the mathematics of precisely-defined points, functions, sets, probability measures, etc. — for coping with the analysis of biological systems, and that to deal effectively with such systems, which are generally orders of magnitudes more complex than man-made systems, we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities which are not described in terms of probability distributions. . . ([76], p. 857).

Since the patient as the subject of medicine is an animate system, medical professionals in research and practice are indeed animate system theorists and “need a radically different kind of mathematics. . .” you have created. The prospect above was concretized later by explicit demonstrations such as “Biological application of the theory of fuzzy sets and systems” [79], and by numerous encouragements and hints we have received from you regularly in your ground-breaking publications. Unfortunately, initial misunderstandings on the nature of fuzzy theory disseminated by some scholars such as Rudolf Kalman, William Kahan and Myron Tribus have been a major obstacle to the awareness in medicine that this theory is no mere addition to the traditional methods and methodologies, but a novel *Weltanschauung* steadily and quietly bringing about a radical revolution of scientific thinking, reasoning and concept and theory formation on the one hand, and of technology and society, on the other. In contrast to ineffective attempts in the past by different scholars to criticize or refute one or the other of the Aristotelian principles in isolation, it successfully terminates the whole Aristotelian paradigm that has been reigning over scientific reasoning and human culture for the last 2300 years. In this sense, it represents a unique, unprecedented example of Thomas Kuhn’s account of scientific change by paradigm shift [33].

The all-embracing paradigm shift caused by fuzzy theory that we are excitedly witnessing is too far-reaching to allow any of the Aristotelian foundations to survive. It, thus, exercises an unfuzzy break with a long-standing and deeply entrenched tradition. To assess the impact of this departure on human culture and civilization in general, and on medicine in particular, we will have to carefully consider what in this process is being replaced with what.

2. Goodbye to the Aristotelian *Weltanschauung*

The Polish physician Ludwik Fleck published in 1935 a remarkable book in German entitled *Entstehung und Entwicklung einer wissenschaftlichen Tatsache*, i.e. genesis and development of a scientific fact, which was neglected by his contemporaries [19,20]. Like Karl Popper’s influential monograph on ‘The Logic of Scientific Discovery’ launched in the same year [40], the Vienna Circle’s logical-empiricist view on the nature of scientific

knowledge was its target.² Unlike Popper and by a detailed historical-philosophical analysis of research on syphilis, however, Fleck developed in his book a fascinating, original, relativistic epistemology according to which scientific knowledge is constructed by a ‘thought collective’, which we would call ‘scientific community’ today, acting in the realm of a particular ‘thought style’. Viewed from within different thought styles, things look different [19–22].

As Thomas Kuhn briefly acknowledges in the foreword to his acclaimed book on the structure of scientific revolutions ([33], pp. vi–vii), Fleck’s widely unrecognized publication has served as the main source of inspiration of his own paradigm-shift theory. Kuhn’s paradigms are in fact Fleck’s thought styles. Since Kuhn had used his notion of a paradigm extremely loosely and received various, incisive criticisms [37,64], he later substituted for it the notion of a ‘disciplinary matrix’ [34]. The disciplinary matrix in a particular scientific area is in essence the set of shared basic rules, methods, and beliefs which underlie theory formation and knowledge acquisition in this area.

Although all of these concepts are still too vague and inadequate to be very useful,³ we may, nevertheless, learn from these studies that in contrast to our accustomed views on the development of science and scientific knowledge, this very development is not a cumulative process. Science does not progress continuously and by accumulating knowledge. It does not add to an antecedent knowledge or theory T_i a subsequent knowledge or theory T_{i+1} of the same type such that one could reasonably consider science as the open, ordered series of related theories T_1, T_2, \dots, T_{i+1} . Scientific ideas, theories, and worldviews evolve discontinuously in that a body of knowledge or theory T_i , which is held over a particular period of time, is dislodged by another body of knowledge or theory T_j because the disciplinary matrix within which the former theory T_i had grown, changes to another disciplinary matrix which gives rise to the new theory, T_j , that is incompatible and incommensurable with its predecessor T_i . For example, the Hippocratic and Galenic humoral pathology rooted in the pre-anatomical era of antiquity considered illness as an imbalance of four humors in the body, i.e. bile, phlegm, blood and urine, and lasted until the eighteenth century. After Andreas Vesalius’ anatomy, *De Humanis Corporis Fabrica* (1543), and the then-emerging early empiricism conceptualized by Francis Bacon and John Locke had made a novel, empirical-anatomical disciplinary matrix available within which illness appeared to have something to do with solid parts of the body, humoral pathology was replaced with the localized pathology of *De Sedibus et Causis Morborum* (1761) by Giovanni Battista Morgagni. By the end of the eighteenth century, it was complemented by

²The Vienna Circle (1923–1936) was a small group of philosophers, natural scientists, social scientists, and mathematicians such as Moritz Schlick, Otto Neurath, Rudolf Carnap, Hans Hahn, Friedrich Waismann, Herbert Feigl, Karl Menger, Kurt Gödel and a few additional ones. It constituted the germ cell of a philosophical movement dubbed ‘logical empiricism’ propagating the view that knowledge of the world is perception plus logic (cf. [9,32]). Ludwik Fleck (1896–1961) was a Polish physician and microbiologist and lived in Lwow, Poland. In June 1941, he was deported to the Jewish ghetto in his city and later, in December 1943, to the concentration camp Buchenwald in Germany to do research on typhus serum. He survived and returned to Poland in 1945 and emigrated to Israel in 1957. He is widely recognized today as the founder of the social constructionist theory of science. For a comprehensive account of his life and work, see [11].

³For an extensive and inspiring explication and elucidation, see [65].

Francois Xavier Bichat's tissue pathology. After the development of the microscope had enabled Theodor Schwann to discover the animal cell around 1838, localized pathology was replaced with Rudolf Virchow's *cellular pathology* (1858), which considered diseases as cellular changes and disorders. With some alterations and additions, this view has been dominating medicine since. We are currently witnessing the emergence of a competing *molecular pathology*, e.g. genomics and pathobiochemistry, which explains and treats diseases as molecular processes in the body. Maybe our descendants will encounter *quantum pathology* or something like that in the near future [59].

In the course of such a discontinuous evolution, a preceding theory T_i is abandoned and a subsequent theory T_j is used instead. This is so not because T_i was demonstrated to be false or T_j was shown to be true, but because concepts and methods have changed which are employed in constructing and acquiring knowledge, i.e. theory T_j in the present case.

Our examples above and also comparable cases dealt with in the literature, such as Newtonian mechanics or Einstein's theory of relativity and the like, exemplify only particular theories and singular pieces of knowledge, each of which emerges from within a specific disciplinary matrix. Interestingly, however, there are also disciplinary matrixes of higher generality, such as the concept of two-valuedness, which are responsible not only for the emergence of individual theories, but for the very mode of scientific thinking and inquiry in all fields. At the highest level of generality we presently encounter, to our surprise, a particular disciplinary matrix which has been nourishing all sciences and theories for the last 2300 years, i.e. *the Aristotelian disciplinary matrix*, because it contains the two-valued, classical logic with which researchers reason and defend their work. What is being eradicated by fuzzy theory is just *this* universal disciplinary matrix. One may, therefore, easily imagine what may happen when this deeply entrenched Aristotelian *thought style* will be replaced with the thought style of fuzzy theory. No conceptual structure, method, knowledge, theory, and research program will survive which does not accord with the new disciplinary matrix.

The foundations of the Aristotelian disciplinary matrix had been laid by Aristotle himself in his *Metaphysics*, *Organon*, and *De Interpretatione*. They would constitute over more than two millennia the basic principles of classical reasoning in science, mathematics, philosophy, religion, politics, law, ethics, and all other areas. The following six passages A–F stated in the *Metaphysics*, together with a seventh concept, G, a modern derivative from his logic, sufficiently represent these principles:

(A) This will be plain if we first define truth and falsehood. To say that what is is not, or that what is not is, is false; but to say that what is is, and what is not is not, is true (*The Metaphysics*, Book IV, 1011 b 26–27) [3].

(B) By demonstrative I mean. . . , e.g. “everything must be either affirmed or denied”, and “it is impossible at once to be and not to be” (*ibid.*, B III, 996 b 27–30).

(C) . . . that is the most certain of all principles. Let us next state what this principle is. “It is impossible for the same attribute at once to belong and not to belong to the same thing and in the same relation” (*ibid.*, B IV, 1005 b 19–23).

(D) Nor indeed can there be any intermediate between contrary statements, but of one thing we must either assert or deny one thing, whatever it may be (*ibid.*, B IV, 1011 23–24).

(E) Further, an intermediate between contraries will be intermediate either as grey is between black and white, or as “neither man nor horse” is between man and horse (ibid., B IV, 1011 b 29–32). . . . Again, there will also be an intermediate in all classes in which the negation of a term implies the contrary assertion; e.g. among numbers there will be a number which is neither odd nor not-odd. But this is impossible. . . (ibid., B IV, 1012 a 8–11).

(F) Again, unless it is maintained merely for argument’s sake, the intermediate must exist beside all contrary terms; so that one will say what is neither true nor false. And it will exist beside what is and what is not; so that there will be a form of change beside generation and destruction (ibid., B IV, 1012 a 5–8).

(G) A set of premises logically implies a conclusion if, and only if, whenever the premises are true the conclusion is true. Passage A is the correspondence concept of truth providing the basis for the rest, and being the root of the correspondence theory of truth and of Tarski semantics of classical, two-valued logic. Passage B is the principle of two-valuedness. The second half of this passage and passage C are alternative formulations of the law of non-contradiction, which says that a statement of the form $\alpha \wedge \neg\alpha$ is contradictory, never true and should therefore be rejected, $\neg(\alpha \wedge \neg\alpha)$. Passages B and D give different forms of the law of excluded middle, $\alpha \vee \neg\alpha$, which say that such a statement is a tautology and always true.

In passage E, two-valuedness is defended once again because otherwise one would have to suppose that there are — sic! — classes without sharp boundaries between members and non-members. But this is, Aristotle says, impossible. So he may be viewed as the progenitor of Georg Cantor’s two-valued set theory.

Passage F in the same vein rejects the view that there may be an intermediate between being and nonbeing. This commonly held Aristotelian ontology will be referred to below as the doctrine of crisp existence. All of us share this ontological doctrine, since all of us believe and assert that ‘everything is or is not, there is no intermediate between being and nonbeing’.

The closing definition, G, of the concept of logical implication, consequence, or inference, has emerged from the modern reconstruction, completion, and axiomatization of Aristotelian logic in the 19 and 20th centuries by Bernhard Bolzano, Gottlob Frege, Bertrand Russell and others. It represents the basic concept of the theory of deduction and proof underlying all classical mathematics, science, and technology.

There have been various attempts in the past to invalidate one or another of the principles or concepts A–G above in isolation. None of them has attained general acceptance, however. For example, many-valued logics run counter to the principle of two-valuedness. Intuitionistic logic does not accept the law of excluded middle. Paraconsistent logics reject the law of non-contradiction.⁴

⁴Paraconsistent logics are inconsistency tolerant systems of logic which do not contain a principle of non-contradiction. They originated around 1910 with the Russian physician Nikolaj A. Vasiliev (1880–1940), who at the beginning of the 20th century taught philosophy at the University of Kazan, Russia. Inspired by Nikolaj Lobachevski’s non-Euclidean geometries in which the Euclidean parallel postulate is not valid, he attempted to construct new, ‘Imaginary Logics’ by discarding some of the basic laws of classical logic [4,70,71]. These logics would enable us to study a large class of ‘imaginary worlds’ that are impossible to classical logic, but nevertheless, quite well imaginable. After Stanislaw Jaskowski’s interlude [29], specific research in this new field of non-classical logics was initiated by the significant work of the Brazilian logician and philosopher Newton C.A. da Costa [12,13]. The term ‘paraconsistent logic’ was coined by the Peruvian philosopher F. Miro Quesada in 1976. For a comprehensive account of the subject, see [25,41,50].

However, the lethal collapse of the Aristotelian disciplinary matrix is caused by fuzzy theory in that all fundamental principles and concepts A–G above are being removed at once in the following way.

The treatment of *truth* in fuzzy theory as a many-valued linguistic variable with a colorful and invigorating term set such as {true, not true, very true, completely true, more or less true, fairly true, false, very false, . . . , etc. . . .}, and the treatment of these terms as labels of fuzzy sets over the unit interval (see [85–87]), is an ingenious and highly esthetic dethronement of all existing theories of truth and of all simplistic semantics, including Aristotele's, Tarski's, Carnap's, and Kripke's perspectives. It goes without saying that whenever the simplistic concept of truth is lost, everything dependent will also vanish. That means that following the fall of A above, B–G will automatically collapse. Fortunately, there is a complete substitute for all of that, the fuzzy theory, which is capable of reigning immediately as the new disciplinary matrix. Its availability as a more than perfect substitute is, thus, the reason of its success.⁵

As a substitute, the fuzzy theory is a many-valued conceptual system. As stated in B above, a consequence of this removal of the Aristotelian two-valuedness is that the principles of excluded middle and non-contradiction, B–D above, are also passé because the union of a fuzzy set X and its complement X^c is not necessarily the base set, and their intersection is not necessarily empty, i.e. $X \cup X^c \neq \Omega$ and $X \cap X^c \neq \emptyset$.

It took approximately 2300 years to refute the Aristotelian position against fuzziness stated in E above and to demonstrate that there are, first, intermediates between members and non-members of a class, and second, numbers which are neither odd nor not-odd, e.g. the fuzzy number 'approximately 2300'.

Thus far, the overall effect is an implosion of the classical concept of two-valued inference sketched in G above and, as a consequence, the decease of all two-valued logics. We will return to this point in Section 4. Let us first supplement the new paradigm with a novel facet, invalidating F above, to complete our goodbye to A–G above.

3. Fuzzy ontology

As already indicated, the human reason seems to hold a two-valued ontology reflected in the common belief that 'everything is or is not, there is no intermediate between being and nonbeing'. This Aristotelian doctrine of crisp existence cited in F above that has governed Western philosophy, logic, science, religion, and commonsense for the last two millennia, no longer applies. To sketch this outlook we will add to the class of fuzzy quantifiers the fuzzy existence operator "there is to some extent", denoted by \exists' . The sentence $\exists'x(Px)$ thus reads *there is to some extent an x such that x is P* . Let P be any predicate that signifies a corresponding set P with the membership function μ_P . Then we may define:

⁵ Mere criticisms of a disciplinary matrix extended from within an empty hole will never suffice to force a shift because the question will arise: a shift to where? For example, we have as yet encountered many such vacuous attempts in medicine, e.g. homeopathy and psychosomatics, which have been vainly trying to shift modern medicine from its natural–scientific concepts and methods to: where? Nobody knows.

Definition 1. $\exists'x(Px)$ if and only if $\exists r(\mu_P(x) = r)$.

An object x whose graded existence is asserted by a statement of the form $\exists'x(Px)$, is a *fuzzy object*. An example is David. He is young to the extent 0.7. Thus, the definition above implies that *there is to some extent someone* who is young. The proposition that to the extent 0.2 the earth is a big planet provides a second example, i.e. the fuzzy object *the earth*. Elementary particles of quantum physics are additional examples which Werner Heisenberg has tried to grasp by his uncertainty principle.

Obviously, any fuzzy set defines fuzzy objects. The fuzzy existence of such an object may be measured by a new operator ${}_P\exists'_r$ that is read ‘relative to P there is to the extent r ’. Accordingly, the expression ${}_P\exists'_r x(Px)$ means that *relative to P there is to the extent r an x such that x is P* . In place of Definition 1, this metric existence operator may be defined as follows:

Definition 2. ${}_P\exists'_r x(Px)$ if and only if $\exists r(\mu_P(x) = r)$.

With regard to our first example above which said that David is young to the extent 0.7, this new concept implies that relative to the predicate *young* there is to the extent 0.7 someone who is young, i.e. David.

According to this new concept, a fuzzy object exists only relative to a particular predicate and to a particular extent between 1 and 0. The transition from being to nonbeing and vice versa is thus gradual rather than abrupt. *Παυτα ρει*. Everything exists to an extent $r \leq 1$. If this $r = 0$ for all predicates of a particular language, the object does not exist with respect to this language. However, it may exist with respect to another language. That means, first, that a language induces an ontology, and second, that being and nonbeing is relative to languages and logics. “Change your language or logic, and you will see another world” ([48], p. 171).

This amounts roughly to saying that to be is to be a fuzzy object, i.e. a member of a fuzzy set within a particular language [50].⁶

4. Fuzzy proof theory

It has been said above that within the new disciplinary matrix the two-valued concept of classical–logical inference implodes. As a result, classical logic ceases to serve as a means of reasoning in science and technology. This decrease is not only due to the above-mentioned implosion. It is also caused by the unparalleled power of *fuzzy logic* to cope with vagueness in the widest sense, i.e. with partially true statements, fuzzy connectives, fuzzy predicates, predicate modifiers, and fuzzy quantifiers.

⁶ Viewed from this perspective, Quine’s well-known account of ontology which says that “to be is to be the value of a variable” ([42], p. 15) appears somewhat simplistic. In developing my ontology over the years, I have profited from discussions with my friend Professor Newton C. Affonso da Costa, University of Sao Paulo, and from his publications and manuscripts, e.g. [14].

Capable of dealing with these and other vague ingredients of natural languages and commonsense reasoning, fuzzy logic started with the doctrine of being a logic of approximate reasoning based on rules of inference whose validity is approximate rather than exact (cf. [83–87,90,5]). Although it has become a truly strong theory of reasoning and control in the meantime, its initial doctrine continues to be widely misunderstood. Many traditional logicians and mathematicians are still hesitant about taking a logic seriously that relies on approximate rules of inference (cf., e.g. [27,28]). This is so because it has not yet been properly ascertained what is actually brought about by fuzzy logic. One considers its surface structure and overlooks the processes in its deep structure. In the depth of fuzzy logic the following development may be observed which represents a philosophically most intriguing aspect of the revolution.

We have already stated that the Aristotelian two-valuedness is being replaced with the fuzzy-theoretical many-valuedness. This substitution is vividly reflected in the fuzzy-theoretical treatment of *truth* as a many-valued linguistic variable. However, this many-valuedness not only concerns *factual* truths and falsehoods when the truth or falsehood of a statement such as, for example, ‘snow is white’ is considered, but also affects the concept of *logical truth* that characterizes the metalinguistic relation of classical-logical inference sketched in G in Section 2. That means that in the depth of fuzzy theory there is a hidden, metalinguistic concept of many-valuedness, i.e. *inferential many-valuedness*, that will more and more emerge as a *fuzzy proof theory* in the near future. With this proof theory at its disposal, fuzzy theory will conquer all mathematical sciences. To shed some light on this imminent course, consider the following, prototypical fuzzy-logical inference:

$$\begin{array}{ll} a \text{ is small} & \equiv \text{statement } \alpha_1 \\ \frac{a \text{ and } b \text{ are approximately equal}}{b \text{ is more or less small}} & \equiv \text{statement } \alpha_2 \\ & \equiv \text{statement } \beta \end{array}$$

Inferences of this type are usually based upon rules such as compositional rule of inference or generalized modus ponens or something else, and are commonly viewed as imprecise. And since what is imprecise is not good, fuzzy logic is not good. A verdict of this kind is strange and wrong simply because it evaluates a fuzzy inference as above from the outmoded perspective of two-valued logic where a statement β either definitely is, or definitely is not, a consequence of a set $\Sigma = \{\alpha_1, \dots, \alpha_n\}$ of premises:

A set $\Sigma = \{\alpha_1, \dots, \alpha_n\}$ of premises classical-logically implies a conclusion β iff whenever the premises are true β is true.

According to this classical-logical concept of inference, the set of all consequences of the premises Σ , denoted by $\text{cons}(\Sigma)$, is the set of all statements that are classical-logically implied by Σ , i.e.

$$\text{cons}(\Sigma) = \{\beta \mid \Sigma \text{ classical-logically implies } \beta\}$$

The set $\text{cons}(\Sigma)$ is crisp. This means that given a set Σ of $n \geq 1$ premises $\alpha_1, \dots, \alpha_n$, traditionally a particular statement β is considered to be implied by Σ if it is true that $\beta \in \text{cons}(\Sigma)$, while it is considered not to be implied by Σ if it is true that $\beta \notin \text{cons}(\Sigma)$. And since it is not evident if a statement such as ‘ b is more or less small’ above is or is not an

element of the set $\text{cons}(\{a \text{ is small, } a \text{ and } b \text{ are approximately equal}\})$, the inference is labeled ‘imprecise’, ‘vague’, or ‘uncertain’. A remedy for this perpetual mistake is provided by Zadeh’s fuzzifiability principle:

Any theory, X , may be fuzzified by replacing the concept of a set in X by the more general concept of a fuzzy set ([98], p. 816).

Thus, let us fuzzify the *set of consequences* of a statement set, $\text{cons}(\Sigma)$. This task may be accomplished by constructing a membership function $\mu_{\text{cons}(\Sigma)}$ that assigns a number $\mu_{\text{cons}(\Sigma)}(\beta) \in [0, 1]$ to a statement β as the degree of its belonging to the set $\text{cons}(\Sigma)$. As a result, we obtain a concept of graded inference that we also call graded logical implication, graded deduction, or graded reasoning (cf. [45]).

Let S be the set of all syntactically admissible sentences of a particular language. A *crisp logic* L over this language is a mapping of the form

$$\Rightarrow_L: 2^S \times S \rightarrow \{0, 1\}$$

with \Rightarrow_L being its implication operator and 2^S the powerset of S such that for all $\Sigma \in 2^S$ and $\beta \in S$ we have $\Rightarrow_L(\Sigma, \beta) = 1$ if according to logic L the statement β follows from the statement set Σ , and $\Rightarrow_L(\Sigma, \beta) = 0$, otherwise. By contrast, a *fuzzy logic* FL over that language is a generalized mapping of the form

$$\Rightarrow_{\text{FL}}: F(2^S) \times S \rightarrow [0, 1]$$

with $F(2^S)$ being the fuzzy powerset of S such that $\Rightarrow_{\text{FL}}(\Sigma, \beta) = r$ says that according to logic FL the statement β follows from Σ to the extent $r \in [0, 1]$. The availability of such a fuzzy implication operator, \Rightarrow_{FL} , would enable us to define our envisaged membership function $\mu_{\text{cons}(\Sigma)}$ above as follows:

$$\mu_{\text{cons}(\Sigma)}(\beta) = r \text{ iff } \Rightarrow_{\text{FL}}(\Sigma, \beta) = r.$$

And that means

$$\mu_{\text{cons}(\Sigma)}(\beta) \Rightarrow_{\text{FL}}(\Sigma, \beta).$$

The extent to which a statement β is a member of the consequences of Σ is, thus, the degree of its deducibility from Σ . We would in this way obtain the fuzzy set $\text{cons}(\Sigma)$ of consequences of the premises Σ :

$$\text{cons}(\Sigma) = \{(\beta, r) | r = \mu_{\text{cons}(\Sigma)}(\beta)\} = \{(\beta, r) | r \Rightarrow_{\text{FL}}(\Sigma, \beta)\}.$$

What fuzzy logic is actually doing will give birth just to such a graded concept of deducibility creating a *quantitative logic* that may be called a fuzzy proof theory. A promising approach to this goal is the interpretation of the rules of fuzzy inference as rules of fuzzy constraint propagation [97,100].⁷

⁷ It is tempting to interpret from this perspective the development of logic as a history of recognizing and handling the phenomenon of fuzzy constraint propagation. The concept of inference in all traditional, deductive logics is a truth preserving relation between premise and conclusion. And that means that all traditional, deductive logics are theories of the propagation of the single constraint “true”. But there are innumerable constraints, including the entire term set $T(\text{truth}) = \{\text{true, not true, very true, quite true, false, } \dots\}$ of the linguistic variable *truth* itself, whose propagation is being studied in fuzzy logic. Also Rudolf Carnap with his probability-based inductive logic may have had some kind of a quantitative logic in mind [10].

While the concept of inference of the quantitative logic above is a numerical variable, a *qualitative* fuzzy logic would emerge from treating the notion of inference as a linguistic variable. For example, let the sentence ‘ Σ implies β ’ be written in prefix notation as $\text{implies}(\Sigma, \beta)$. Here, ‘implies’ is a binary linguistic variable. A possible term set for this linguistic variable, $T(\text{implies})$, is exemplified by the following arrangement:

$$T(\text{implies}) = \{\text{absolutely, strongly, quite strongly, very strongly, not strongly, weakly, very weakly, } \dots \text{ etc. } \dots \}.$$

In clinical medicine, for instance, a qualitative fuzzy logic would enable the physician to judge to what extent the patient’s data implies a particular diagnosis. For example, the result of such a diagnostic reasoning may be:

$$\begin{aligned} \text{implies}(\text{patient_data} \cup \text{KB}, \text{ the patient has diabetes}) &= \text{strongly} \\ \text{implies}(\text{patient_data} \cup \text{KB}, \text{ the patient has hepatitis}) &= \text{very weakly} \end{aligned}$$

where patient_data in the premises comprise a collection of statements about the patient’s complaints, symptoms, and signs, and KB is the knowledge base used. Alternatively, the implication relation may also be constructed as ‘ β is implied by Σ ’, expressed by the linguistic variable $\text{is_implied}(\Sigma, \beta)$ with the term set:

$$T(\text{is_implied}) = \{\text{high, medium, low, very high, very low, } \dots \text{ etc. } \dots \}.$$

In this case, the physician may arrive, for example, at the clinical judgment:

$$\begin{aligned} \text{is_implied}(\text{patient_data} \cup \text{KB}, \text{ the patient has diabetes}) &= \text{very high,} \\ \text{is_implied}(\text{patient_data} \cup \text{KB}, \text{ the patient has hepatitis}) &= \text{quite low.} \end{aligned}$$

In either case, we would in clinical reasoning become able to exploit the powerful theories of linguistic variables, information granulation, and computing with words [81,82,85–89,95,97,99,100].

5. Fuzzy-theoretical worldmaking

The development and application of fuzzy theory is steadily accelerating. There is no doubt that in a few years the theory will constitute the universal disciplinary matrix in all areas of science, humanities, technology, and human reasoning. The three main pillars upon which it was initially built and still rests and grows are fuzzy set theory, the theory of linguistic variables, and fuzzy logic. As was already referred to in Section 1, the motivation behind this innovation has been the perception that for coping with the analysis of highly complex systems such as the animate ones “we need a radically different kind of mathematics, the mathematics of fuzzy or cloudy quantities...” ([76], p. 857).

As a mathematically oriented system theorist, I had been conditioned to believe that the analytical tools based on set theory and two-valued logic were all that was needed to build a framework for a precise, rigorous and effective body of concepts and techniques for the analysis of almost any kind of man-made or natural system,

including control systems. Then, in 1961–1963, in the course of writing a book on system theory with C.A. Desoer, I began to feel that highly complex systems — typified by economic and biological systems — cannot be dealt with effectively by the use of conventional approaches. My feeling derived, in the main, from a realization that system description languages based on classical mathematics are not sufficiently expressive to serve as a means of characterization of complex input–output relations in an environment of imprecision and uncertainty.

The culprit as I saw it was the universally made assumption that classes have sharply defined boundaries. They do in classical mathematics, but in the real world that we live in the opposite is the case, that is, almost all classes are fuzzy in the sense that the transition from membership to nonmembership in such classes is gradual rather than abrupt. Accepting this fact, the obvious thing to do is to assume that membership in a class is a matter of degree. This assumption is the genesis of the theory of fuzzy sets.

After I wrote my first paper on fuzzy sets in 1965, my aims as a system theorist underwent a marked shift. I came to the conclusion that not just control theory and systems analysis, but, more generally, most or all scientific methodologies will have to undergo a critical reexamination and move toward the replacement of their crisp foundations with foundations based on fuzzy set theory and fuzzy logic, aiming at a greater generality and better approximation to reality. In the realm of control, in particular, the replacement of crisp logic with fuzzy logic could make it possible to design systems with a much higher MIQ (Machine Intelligence Quotient) compared to those which can be designed by traditional methods... ([96], p. 192).

The worldwide implementation and proliferation of the theory in research and practice within a short period of 36 years after its inception is a convincing proof that all of the intuitions above were correct. But characterizations such as “aiming at better approximation to reality” that we encounter in appraisals of fuzzy theory seem to presuppose a ‘reality out there’ that would exist independently of a respective theory from within which ‘that reality’ is viewed. This assumption is worthy of reconsideration because a theory both generates and shapes its own domain of application and development, and thus, ‘the reality’. It provides the categories that are imposed on the basic domain of discourse, the labels to use in categorizing the objects, and the methods for categorizing, describing, and analyzing them. In this sense, it is an a priori frame of reference with a peculiar, new syntax and new semantic basis of language meaning that will influence the subjects and modes of communication and negotiation on the descriptions it will generate. This is especially true of fuzzy theory, since it is at the same time the disciplinary matrix and includes the logic to obey. Once we attempt to describe the ‘reality out there’, we enter a world that is induced by fuzzy-theoretical language itself. For example, in a Georg Cantor world induced by crisp set theory there are no fuzzy sets, no fuzzy numbers, no fuzzy control, no fuzzy expert systems and the like. Even those sets denoted by fuzzy predicates such as *big*, *tall*, *young*, and *red* are handled as crisp sets with clear-cut boundaries. However, furnish the population with the hammers of fuzzy sets, linguistic variables and fuzzy logic, and the world is full of nails. What they were not able to see before, they now discover everywhere. To put it concisely, *the reality out there we see through fuzzy-theoretical*

glasses is fuzzy-theoretically constructed. Facts are made by visions. And “One must know before one can see” [20].

Thus, fuzzy sets as the basis of the whole Fuzzy Revolution are not a discovery, but a *creation*. It is improper to insinuate, as it is done sometimes, that Bertrand Russell and Max Black were the progenitors of this creation, although it is true that these two scholars noticed the problem of vagueness and analyzed the trouble it causes to classical logic [43,6,7]. But it is also equally true that they were not the discoverers of the problem and did not attain any solution (see [60,62]). The problem is an old one that has already been well known in medicine for several centuries. The vagueness and elasticity of terms such as *ill, well, sick, healthy, remedy, placebo, treatment efficacy, recovery, cure, alive, dead, pneumonia, appendicitis, headache, delusion, depression, fever, pain, acute, chronic* and of the rest of medical vocabulary has concerned physicians for a long time. Due to the lack of a creative solution like *Fuzzy Sets* they have come to the sterile conclusion that clinical decision-making is not a science, but an *art* because it is ineluctably faced with vagueness, imprecision, and uncertainty. This soporific view is the sobering reason why, in spite of all scientific progress in biomedicine and medical technology, clinical judgment has remained archaic and still produces about 40% misdiagnoses [26,46,52–54,57].⁸

Our suggestion above that the post-Aristotelian reality is being fuzzy-theoretically constructed may now be understood in the following way.⁹

Let world $W = \{x, y, z, \dots\}$ be a particular collection of any entities such as {apple a, apple b}, and let $D = \{\alpha_1, \alpha_2, \alpha_3, \dots\}$ be any *description* where each α_i is a statement about W , for example, {apple a is red, apple b is very big}. How do we get such a description D about a world W ? That is, where does our worldview on W come from?

A natural language, and any theory as well, may be conceived of as an n -valued linguistic variable of higher order that takes sets of sentences as its values. Such a set is a set $D = \{\alpha_1, \alpha_2, \dots\}$ of *descriptions* the variable generates when at a particular instant of time it is applied to a particular world $W = \{x, y, z, \dots\}$ of any entities. It is, therefore, referred to here as a *description variable* d such that $d(W) = D$.

For example, let the description variable d be the theory of cellular pathology and let world W be a particular bioptic specimen a of human skin. The application of this description variable to this world at a particular instant of time yields a description of the cells of the specimen a . That is, $\text{cellular_pathology}(\text{specimen } a) = \{\text{the cells of the specimen } a \text{ are such and such}\}$.

Thus, to say that a world W is describable by a particular description variable d amounts to saying that there is a description $D = \{\alpha_1, \alpha_2, \dots\}$ such that $d(W) = D$. Analogously, the assertion that a world W is describable by a set of statements, D , is equivalent to the existence postulate that there is a description variable d such that $d(W) = D$.

Definition 3. x is an L -logical implication structure iff there are A, B , and \Rightarrow_L such that

⁸Fuzzy machines will change this situation in that they will allocate to the doctor the role of a mobile peripheral for gathering patient data [59,61]. See Section 8.

⁹For a detailed discussion, see [50].

1. $x = \langle A, B, \Rightarrow_L \rangle$;
2. A and B are non-empty sets of statements;
3. there is a logic L such that \Rightarrow_L is its implication operator;
4. $\Rightarrow_L(A, \beta) \neq 0$ for each statement $\beta \in B$.

This definition says, in essence, that each statement in set B is to a particular extent L -logically implied by set A . For example, the following triple is a classical–logical implication structure: $\langle \{\text{All men are mortal, Socrates is a man}\}, \{\text{Socrates is mortal}\}, \Rightarrow_{\text{classical–logic}} \rangle$. For we have

$$A = \{\text{All men are mortal, Socrates is a man}\},$$

$$B = \{\text{Socrates is mortal}\},$$

$$\Rightarrow_L = \Rightarrow_{\text{classical–logic}}$$

$$\Rightarrow_{\text{classical–logic}}(\{\text{All men are mortal, Socrates is a man}\}, \{\text{Socrates is mortal}\}) = 1.$$

Definition 4. x is a *world description* iff there are W, D , and d such that

1. $x = \langle W, D, d \rangle$;
2. W is a non-empty set of any entities, called a ‘world’;
3. D is a set of statements;
4. d is a description variable such that $d(W) = D$.

For instance, the following triple is a world description: $\langle \{\text{apple a, apple b}\}, \{\text{apple a is red, apple b is very big}\}, \text{ordinary language} \rangle$.

Definition 5. x is a *d-theoretical cum L-logical world* iff there are W, D_1, D_2, d , and \Rightarrow_L such that

1. $x = \langle W, D_1, D_2, d, \Rightarrow_L \rangle$;
2. $\langle W, D_1 \cup D_2, d \rangle$ is a world description;
3. $\langle D_1, D_2, \Rightarrow_L \rangle$ is an L -logical implication structure.

Such a *world* emerges in that the description variable d generates a set D_1 of statements on W as antecedents, and the logic L produces their consequences, D_2 . For example, the following pentuple (1) is a fuzzy set-theoretical cum fuzzy-logical world:

$$\begin{aligned} & \langle \{\text{apple a, apple b}\}, \\ & \quad \{\text{apple a is red, apple b is very big}\}, \\ & \quad \{\text{apple b is big}\}, \\ & \quad \text{fuzzy set theory, } \Rightarrow_{\text{FL}} \rangle. \end{aligned} \tag{1}$$

It is not a circuit-theoretical cum classical-logical, or a moral-theoretical cum deontic-logical world. Set $D_1 = \{\text{apple a is red, apple b is very big}\}$ fuzzy-logically implies each element of set $D_2 = \{\text{apple b is big}\}$.

Given a d -theoretical cum L -logical world of the form $\langle W, D_1, D_2, d, \Rightarrow_L \rangle$ and a d' -theoretical cum L' -logical world of the form $\langle W', D'_1, D'_2, d', \Rightarrow_{L'} \rangle$, we call them equivalent if $W = W'$ and $D_1 = D'_1$ and $D_2 = D'_2$.

Definition 6. W is a *basic world* iff there is a supplement $\langle D_1, D_2, d, \Rightarrow_L \rangle$ such that W is extendible to a d -theoretical cum L -logical world $\langle W, D_1, D_2, d, \Rightarrow_L \rangle$.

With regard to our last example, the pair {apple a, apple b} constitutes a basic world since there is a supplement that may extend it to the fuzzy set-theoretical cum fuzzy-logical world (1) above. Obviously no world equivalent to this one is accessible from that basic world by Newtonian mechanics cum classical logic, circuit theory cum probability calculus, quantum theory cum quantum logic, botany cum intuitionistic logic, and the like.

To give real-world examples, let us call the ordinary world of our common perceptions we live in, our lifeworld W_0 . Now, consider areas such as fuzzy control, fuzzy knowledge engineering, fuzzy neurocomputing, computing with words, fuzzy pattern recognition, fuzzy image processing, or fuzzy technology in general. This technology provides innumerable supplements of the form $\langle D_1, D_2, \text{fuzzy theory}, \Rightarrow_{FL} \rangle$ which enable us to make fuzzy-theoretical cum fuzzy-logical worlds of the structure

$$\langle W_0, D_1, D_2, \text{fuzzy theory}, \Rightarrow_{FL} \rangle$$

out of our lifeworld W_0 . There are no competing supplements providing equivalent worlds. This we call fuzzy-theoretical worldmaking. The post-Aristotelian *reality* is a construct of fuzzy-theoretical worldmaking.¹⁰

Given a worldmaking of the structure $\langle W_0, D_1, D_2, d, \Rightarrow_L \rangle$, its d -theoretical and L -logical apparatus $\langle d, \Rightarrow_L \rangle$ is a *conceptual framework* that provides a description $D_1 \cup D_2$ of the basic world W_0 . Due to differing ontologies they induce (see Section 3), different conceptual frameworks generate different world descriptions which may or may not be equivalent. Usually they are not. In any case, a conceptual framework $cf = \langle d, \Rightarrow_L \rangle$ that by operating upon a basic world W_0 constructs a new world $W = D_1 \cup D_2$ may be viewed as an operator termed a *worldmaking operator* such that $cf(W_0) = W = D_1 \cup D_2$. Disciplinary matrixes, theories, and methodologies are worldmaking operators.

By applying different worldmaking operators it is of course *possible* to make out of a basic world W_0 any of the innumerable, imaginable worlds W_1, W_2, \dots, W_{i+1} which may, therefore, be called *possible worlds*. A possible world W_j is accessible from another world $W_i, i \geq 0$, if there is a worldmaking operator cf such that $cf(W_i) = W_j$. A cf -construct such as W_j is not true or false, but simply *good, better than* or *worse than* another one. Thus, different possible worlds W_i and W_j which are accessible from a particular world W_0 may be rank-ordered according to what they mean to W_0 's inhabitants with respect to a body of shared *values*.

For this reason, the appropriate question to ask in evaluating a conceptual framework as a worldmaking operator is not the ancient question of how much truth it generates, but the 21st century question of to where it may lead us. The ultimate measure of judgment on

¹⁰For a reconstruction of 'reality' as a linguistic variable, see [62]. The inspiration for my 'worldmaking' view has come from Nelson Goodman's works [23,24]. For details, see [50].

conceptual frameworks is not mathematics, logic, epistemology or any veritology, but ethics and esthetics, i.e. not the first Kantian question “What can I know?”, but the second one: “*What shall I do?*”. Why shall I prefer action A to action B to bring about world W_A rather than world W_B ? ([49], p. 16).

From this perspective, the initial hostilities towards fuzzy theory may now appear as symptomatic of an unworldly precisionism that is primarily interested in precision and in producing precise worlds, but not in whether or not they are of any relevance and usefulness to anyone other than their producers.

Reflecting this tradition, Professor Rudolf Kalman — one of the foremost contributors to system theory and control — had this to say about my work in 1972:

I would like to comment briefly on Professor Zadeh’s presentation. His proposals could be severely, ferociously, even brutally criticized from a technical point of view. This would be out of place here. But a blunt question remains: Is Professor Zadeh presenting important ideas or is he indulging in wishful thinking? No doubt Professor Zadeh’s enthusiasm for fuzziness has been reinforced by the prevailing political climate in the US — one of unprecedented permissiveness. ‘Fuzzification’ is a kind of scientific permissiveness; it tends to result in socially appealing slogans unaccompanied by the discipline of hard scientific work and patient observation. Let me say quite categorically that there is no such thing as a fuzzy scientific concept, in my opinion.

In a similar vein, a colleague of mine, Professor William Kahan, commented in 1975:

Fuzzy theory is wrong, wrong, and pernicious. I can not think of any problem that could not be solved better by ordinary logic. What Zadeh is saying is the same sort of things ‘Technology got us into this mess and now it can’t get us out’. Well, technology did not get us into this mess. Greed and weakness and ambivalence got us into this mess. What we need is more logical thinking, not less. The danger of fuzzy theory is that it will encourage the sort of imprecise thinking that has brought us so much trouble ([98], p. 812).

Strict adherence to precisionism regardless of its consequences is due to a long-standing laissez-faire and asocial ethics in science. Scientists are not taught to consider and evaluate the social significance of their own actions. It is well known that usually their ethics is confined to the publish or perish principle. For example, a vast number of quantitative inquiries have been and continue to be made in psychology and medicine into human behavior, human diseases, human body and physiology, etc. In a world abundant in poverty and suffering the researchers and research funding agencies waste human resources while neglecting the fact that their harvests fade away within the same year merely on the ground that competing investigations report different results, e.g. ‘the intensity of the variable X in the population Y is not 0.003, but 0.003001’.

Given the deeply entrenched tradition of scientific thinking which equates the understanding of a phenomenon with the ability to analyze it in quantitative terms, one is certain to strike a dissonant note by questioning the growing tendency to analyze the behavior of humanistic systems as if they were mechanistic systems governed by difference, differential, or integral equations... Essentially our con-

tion is that the conventional quantitative techniques of system analysis are intrinsically unsuited for dealing with humanistic systems or, for that matter, any system whose complexity is comparable to that of humanistic systems. The basis for this contention rests on what might be called the *principle of incompatibility*. Stated informally, the essence of this principle is that as the complexity of a system increases, our ability to make precise and yet significant statements about its behavior diminishes until a threshold is reached beyond which precision and significance (or relevance) become almost mutually exclusive characteristics. It is in this sense that precise quantitative analyses of the behavior of humanistic systems are not likely to have much relevance to the real-world societal, political, economic, and other types of problems which involve humans as individuals or in groups ([81], p. 28).

6. The ontology of fuzziness

Despite the skeptics' resistance it is more and more being acknowledged that fuzziness certainly exists. But there is still some controversy about whether it is a subjective characteristic of human beings or an objective feature of the 'reality out there'. Now that fuzzy theory has come of age, the fruitless debate on this naïve question may be closed. Fuzziness is ubiquitous. Open your eyes, and you will see it everywhere.

Interestingly, the incompatibility principle above combines both aspects of fuzziness, its subjective and objective sources, in a single statement. We will demonstrate that the statement belongs to the same class as the Heisenberg uncertainty principle $\Delta p \cdot \Delta q \geq h/2\pi$. Like the latter, it conjoins two incompatible variables and has the structure of a Cauchy–Schwarz inequality $a \cdot b \geq c$.¹¹

To this end, let us first arrange some terminology. Given any object 'sys' of any complexity and any individual x , we write:

$\text{pre}(\text{sys}, x) \equiv$ degree of *precision* of statements about sys made by x
 $\text{sig}(\text{sys}, x) \equiv$ degree of *significance* of statements about sys made by x
 $\text{compl}(\text{sys}) \equiv$ degree of *complexity* of sys
 $\omega \equiv$ a constant (see below).

Our reconstruction of the principle is the following *Zadeh incompatibility principle 1*:

$$\frac{1}{\text{pre}(\text{sys}, x)} \times \frac{1}{\text{sig}(\text{sys}, x)} \geq \frac{1}{\text{compl}(\text{sys})} \omega \quad (\text{Zadeh IP-1})$$

Since all of us wish 'always to make maximally significant statements' about a system that we study, hold $\text{sig}(\text{sys}, x)$ constant at an appropriate level. First, whenever $\text{compl}(\text{sys})$ increases, and thus, lowers the fraction at the right hand side, the increase in $\text{pre}(\text{sys}, x)$

¹¹ Here we have only presented the formal skeleton of the Cauchy–Schwarz inequality. For an interesting and inspiring interpretation of the Heisenberg uncertainty principle as a Cauchy–Schwarz inequality, see ([31], p. 114). There are also other interesting and inspiring interpretations, e.g. [66,67].

must be limited and even reversed to maintain the inequality. That is, the compatibility between complexity and precision is limited. Second, in the case of constant $\text{compl}(\text{sys})$, precision $\text{pre}(\text{sys}, x)$ and significance $\text{sig}(\text{sys}, x)$ are incompatible in that they cannot be simultaneously maximized. Increasing precision lowers significance and vice versa.

Note that the Zadeh IP-1 is a heuristic observation about human ability in coping with complexity. It is not an empirical–statistical statement like the Heisenberg uncertainty principle. But it may be tested empirically. The functions pre and sig would then have to be interpreted as random variables to measure their variances in populations of human observers. Also the constant ω could then be empirically identified.

In essence, Zadeh IP-1 implies a second, informal version that may be expressed as follows:

In a world of high complexity precision is undesirable if significance is desired
(Zadeh IP-2)

By substituting ‘fuzziness’ as an antonym for ‘precision’ in IP-2 we obtain a Zadeh compatibility principle:

In a world of high complexity fuzziness is desirable if significance is desired
(Zadeh CP)

The last two Zadeh principles are conditional preference rules implicitly underlying fuzzy theory. They demonstrate that the question of whether or not fuzziness exists objectively and independently of human beings, is irrelevant. To comply with these principles, one will just *make* fuzziness to cope with the increasingly complex world. What is needed is only to know what fuzziness is. What is fuzziness? This query also need not give rise to controversial philosophies since there is a simple and uncontroversial answer, that is ‘see Zadeh [77,78]’. In these twin papers, fuzziness came into being in 1965 by a *definition* as a basis of a magnificent theory and worldmaking operator in the sense discussed above. Thus, as already was stressed in the preceding sections, it is a creation and not a discovery of a pre-existent entity. The ancillary conceptual framework has sometimes been misinterpreted obscuring its essential aspect which may be identified as follows: the *definition* subsumes a particular kind of structure under the label ‘fuzzy set’, whether they be abstract or concrete, real or unreal, natural or artificial, subjective or objective, existent or non-existent. We may shed some light on this freedom of the creator of a class by introducing the notion of a *basic Zadeh structure*:¹²

Definition 7. ξ is a *basic Zadeh structure* iff there are X, f , and A such that

1. $\xi = \langle X, f, A \rangle$;
2. X is a non-empty set;
3. f is a function such that $f: X \rightarrow [0, 1]$;
4. $A = \{(x, f(x)) | x \in X\}$.

¹²For a more general concept of a Zadeh structure, see [55].

Definition 8. A is a *fuzzy set* iff there are X and f such that $\langle X, f, A \rangle$ is a basic Zadeh structure.

Concerning the ontological question of whether or not there are fuzzy sets, Definition 8 above is absolutely neutral and does not claim or imply anything. It does something more important, however. It *determines* the post-definitional era in that anything in the ‘reality out there’ that may accord with the definition will be categorized as a fuzzy set contributing to the assertion that *there are fuzzy sets*. But this being is not independent of the concept creator because in his definition he requires a function f that according to Definition 7 maps the base set X to the unit interval. Functions and mappings do not exist in the ‘reality out there’. They are designed by human beings. The fuzzy-theoretical description of the ‘reality out there’ and the patterns delineated therein thus appear as human constructs. Descriptum ipsum constructum.¹³

The entire Fuzzy Revolution originates in a definition that of course is rooted in the pre-definitional life and Weltanschauung of a genius. Since the theory that was built around the definition was able to successfully cope with highly complex systems in an environment of imprecision, uncertainty, and vagueness, it proved to be a successful worldmaking operator. And since *the patient* also is a highly complex system in an environment of imprecision, uncertainty, and vagueness, medical sciences and clinical decision-making are one of the prominent application domains of this operator.

7. Fuzzy theory in medicine

Biomedical research, so-called, comprising cytology, physiology, biochemistry and other fields usually conducted on animals, is in fact zoology and *paramedical auxiliary* and should not be mistaken for medicine. It is true that this auxiliary paramedical research is based on natural–scientific principles of inquiry and is, therefore, a natural science discipline with its own methodology.

Medicine as a healing profession, however, is not a natural science discipline. It is concerned with the health, illness, disease, therapy, life, and death of the *patient* as a human being, i.e. with something that is defined not by nature, but by human values, society, and culture. Accordingly, the statements that it produces and which control the behavior of the physician in diagnostic–therapeutic decision-making, are in the main conditional imperatives of the form ‘If A is B_1 and . . . and Z is B_m , then *do* C_1 & . . . & C_n ’ where $m, n \geq 1$ (cf. [49,52,58]). Insofar as medical thinking and practice has been concerned with this value-laden and action-theoretical subject rather than with zoology, it has taken place in a methodological vacuum until now. Medical students, doctors, and scientists have never been taught a methodology for their clinical decision-making and research simply because there is as yet no such methodology in medicine [44,52,59]. A major obstacle to its emergence and development has been the fact that medical language and knowledge are inherently and irremediably vague and, therefore, not amenable to

¹³ Alluding to Giambattista Vico’s dictum “verum ipsum factum” [72].

traditional methodological approaches that rely on precisionism. Consider, for example, the following description of pneumococcal lobar pneumonia:

In adolescents and adults the onset is sudden and may come ‘out of the blue’; but often the patient has a cold or other upper respiratory infection and rapidly becomes much more ill, perhaps with an initial rigor but always with a sharp rise in temperature, usually to 101–103°F. Pleuritic pain usually develops over the affected lobe. The patient may become aware that he is breathing rapidly and certainly feels ill. Initially there may be a dry, painful cough but soon the cough becomes productive of sputum which is characteristically ‘rusty’ due to its content of altered blood from the foci of red hepatization; quite commonly, however, it is purulent or slightly bloodstained. It is often viscid and difficult to expectorate and this adds to the patient’s pain ([39], p. 18.28).

This passage from a standard medical textbook exemplifies the usual medical language and knowledge. Replete with vague, natural language terms such as *adolescent*, *adult*, *sudden*, *often*, *cold*, *rapidly*, *ill*, *much more ill*, *perhaps*, *rigor*, *usually* and so on, it conspicuously demonstrates that medicine is not mathematical physics or mathematical biology. It is an inexact action field because, first, the language and knowledge of the subjects constituting this field, i.e. the health care personnel and the patient, are inexact and uncertain, and second, their goals and decisions based upon that language and knowledge are imprecise and uncertain as well. Fuzzy theory has enabled us to view *this* ubiquitous vagueness and uncertainty in medicine as an unavoidable consequence of the complexity and continuity of the ‘reality out there’, and to learn how to cope with it.

Seen from this new perspective, the patient as the subject of medical language, knowledge, goals, and decisions appears as a highly complex bio-psycho-moral system that is primarily governed by continuous variables [61]. Thanks to Zadeh incompatibility principles 1 and 2 above we have learned to realize that due to this peculiarity it is *neither possible nor necessary* to make precise every medical term and decision and thereby awkwardly make discrete the given continuum. On the contrary, the Zadeh compatibility principle CP above suggests that it is even desirable to fuzzify it, since significance is highly desired in medicine. This task is easily attainable in the following way.

The denotation of a medical term is a class of any objects or processes. The Zadeh fuzzifiability principle cited in Section 4 above will always enable us to reconstruct and treat this class as a fuzzy set. It will thus be correct and advantageous to postulate that

Everything in medicine is fuzzy (2)

rendering the entire medicine an application domain of fuzzy theory. We in medicine are grateful to Professor Lotfi Zadeh for having enabled us to establish the basic paradigm (2) and to state accordingly that about 2370 years after medicine’s constitution as a discipline and profession by Hippocrates, finally *habemus methodologiam!* All logical, methodological, and epistemological problems associated with medical vagueness now appear tractable. We may, therefore, also say goodbye to invasive precisionism (cf. [47], pp. 110–111).

Thanks to (2), medicine provides fertile ground for fuzzy theory and its subtheories from fuzzy set theory to possibilistic logic to fuzzy pattern recognition to fuzzy sensors and

automata and additional yet-to-emerge methods and techniques. All of these conceptual frameworks and technologies will serve as a welcome medical intelligence enhancing methodology. Research and practice of the following type have already advanced worldwide and may exponentially increase in the years ahead:

- Studies in the fuzzy foundations of medicine, e.g. concepts of fuzzy health, illness, disease, recovery, therapy, treatment efficacy, etc.;
- Applied fuzzy theory in all fields of medical research and practice, e.g. fuzzy anatomy, fuzzy physiology, fuzzy biochemistry, fuzzy pathology, etc.
- Fuzzy systems theory in medicine, e.g. theories of organism, consciousness, psyche, psychosomatic systems, infection systems, immune systems, etc.;
- Fuzzy signal processing, e.g. EEG, ECG, EMG, ERG;
- Fuzzy monitoring, e.g. in intensive care units;
- Fuzzy adaptive control, e.g. in anesthesia, intensive care units, therapeutic devices;
- Fuzzy image processing, e.g. in radiology, clinical anatomy, and clinical specialties;
- Fuzzy clustering, e.g. in nosology and epidemiology;
- Fuzzy pattern recognition, e.g. in genetics and genomics;
- Fuzzy organ support and prosthesis, e.g. in rehabilitation medicine;
- Fuzzy databases and data engineering, e.g. in hospitals and laboratories;
- Fuzzy analysis and interpretation of laboratory data, e.g. in pathology and clinical-chemistry laboratories;
- Fuzzy sensors in all medical domains;
- Fuzzy medical linguistics and terminology, yet to be developed;
- Fuzzy medical knowledge discovery, yet to be developed;
- Methodology of fuzzy concept and theory formation in medicine, yet to be developed;
- Fuzzy anamnestics, yet to be developed by utilizing branching questionnaires;
- Fuzzy medical knowledge engineering in every medical domain;
- Fuzzy clinical reasoning, e.g. in diagnostic-therapeutic decision-making.¹⁴

An increasingly important role in this evolution will certainly play the latter two subdomains by utilizing the core fuzzy theory, i.e. fuzzy set theory plus linguistic variables plus fuzzy logic to contribute to the development of *fuzzy artificial intelligence*.

8. Fuzzy artificial intelligence: FAI

Once upon a time there was a young doctor of electrical engineering who at the beginning of his scientific career published a paper on ‘Thinking machines’ in *Columbia Engineering Quarterly* half a century ago. His paper began with the following lines:

In the past five years and particularly since the publication of Wiener’s *Cybernetics* (1948), an increasing number of scientists and laymen have become aware of a scientific development which promises to revolutionize not only many fields of science and engineering, but also our whole way of living ([75], p. 12).

¹⁴ See, for example, [1,68], past issues of this journal, and the present volume.

Our young engineer was talking in this prognosis about the then-emerging machine capability that would later be termed ‘Artificial Intelligence’. The replacement of only two words in his statement yields the present keynote:

In the past 39 years and particularly since the publication of Zadeh’s *Fuzzy Sets* (1965), an increasing number of scientists and laymen have become aware of a scientific development which promises to revolutionize not only many fields of science and engineering, but also our whole way of living.

Placed in an engineering journal out of the philosopher’s sight, the young engineer’s article appeared in the same year as Alan Turing’s legendary paper that dealt with the related question ‘Can machines think?’ and was presented in a renowned philosophy journal [69]. While Turing’s affirmative answer to this question has provoked the philosophers’ Chinese Room argument against machine mind [63], their back-to-anthropocentrism slogan “the human mind has the upper hand over any machine” [15,16], and an enduring debate thereon,¹⁵ no philosopher has noticed yet that *thinking machines* are really about to emerge soon from within the non-Turingian AI paradigm of our young engineer. The self-assured anti-AI philosopher may then be surprised at encountering a neurofuzzy *Machina Sapiens* who will be able not only to think, but even to do so much as human beings do, i.e. by computing with words.¹⁶

The term ‘*computing with words*’ is a recent coinage [97–100]. But the goal and method it indicates was envisioned by our young engineer at the very inception of his fuzzy theory, particularly since the *linguistic approach* within which fuzzy logic emerged through treating truth as a linguistic variable [80–88,90,92–94]. As a retreat from the traditional veneration for numerical methods in the face of overpowering complexity, and based on the theory of linguistic variables and hedges, the *linguistic approach* states at the outset the clear “premise that the key elements in human thinking are not numbers, but labels of fuzzy sets” ([81], pp. 28 f.). Complemented later by the theory of granulation of information and the computational theory of perceptions, it has grown into the fascinating theory of computing with words we have recently been presented with [98–100].

Perceptions are usually described by natural language propositions such as “this apple is red” composed of vague *words*. They may, thus, be handled within the linguistic approach and by means of fuzzy logic. To utilize perceptions in the reasoning process and to derive knowledge-based conclusions from their vague descriptions will, therefore, require that they be formulated in, or translated to, a fuzzy logic compatible syntax. This task is accomplished by the Generalized Constraint Language GCL [100]. The GCL is the interface between natural language and fuzzy logic. In this language, a proposition is expressed as a generalized constraint in the sense of fuzzy logic such that inferential relations between premises and conclusions may be construed as constraint propagation from the former to the latter. The rules of constraint propagation coincide with the rules of inference in fuzzy logic. Thus, with reference to computing with words as the recent AI apex of fuzzy theory we may state succinctly that *fuzzy artificial intelligence* is *fuzzy-logic*

¹⁵ See the Kluwer journal *Minds and Machines*, 1991 ff., and [8,17,18].

¹⁶ For a detailed theory of *Machina Sapiens*, see [56]. The name ‘*Machina Sapiens*’ was coined in [51].

programming. More specifically, we may postulate that FAI = perception + fuzzy knowledge base + computing with words.

Give a machine FAI plus self-perception, and you have a thinking machine (for details, see [56]). For example, a thinking machine for clinical decision support so as to reduce misdiagnoses and wrong treatment decisions. Medical knowledge-based and expert systems research in diagnostics and therapeutics may be seen as a contribution to this goal. The perceptions of such a machine may come from the physician who provides it with patient data, or they may originate in its own fuzzy sensors (see [38]).

Our young doctor of electrical engineering has thus guided us to the best way toward the thinking machine that indeed “promises to revolutionize not only many fields of science and engineering, but also our whole way of living”. And what we will render to him for the manifold gifts we have received from his work over the years is our admiration and gratitude and our anticipations for the second phase of The Fuzzy Revolution. The following pages of this volume may demonstrate that we in medicine are starting this second phase. We are pleased and feel honored to dedicate to him our works.

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